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BIANCHI TYPE I ANISOTROPIC MAGNETIZED COSMOLOGICAL MODELS WITH VARYING Λ

ANIRUDH PRADHAN* and OM PRAKASH PANDEY

*Department of Mathematics, Hindu Post-graduate College,
Zamania, Ghazipur 232 331, India*

Bianchi type I magnetized cosmological models in the presence of a bulk viscous fluid are investigated. The source of the magnetic field is due to an electric current produced along x-axis. The distribution consists of an electrically neutral viscous fluid with an infinite electrical conductivity. The coefficient of bulk viscosity is assumed to be a power function of mass density. The cosmological constant Λ is found to be positive and is a decreasing function of time which is supported by results from recent supernovae observations. The behaviour of the models in presence and absence of magnetic field are also discussed.

1. Introduction

The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out Zeldovich et al.[1] Also Harrison [2] has suggested that magnetic field could have a cosmological origin. As a natural consequences we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model.[3] The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors. [4 −13] Strong magnetic fields can be created due to adiabatic compression in clusters of galaxies. Large-scale magnetic fields give rise to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and it decays slower than if the pressure was isotropic.[14,15] Such fields can be generated at the end of an inflationary epoch.[16− 20] Anisotropic magnetic field models have significant contribution in the evolution of galaxies and stellar objects. Bali and Ali[21] obtained a magnetized cylindrically symmetric universe with an electrically neutral perfect fluid as the source of matter. Several authors [22−26] have investigated Bianchi type I cosmological models with a magnetic field in different context.

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*E-mail: acpradhan@yahoo.com, pradhan@iucaa.ernet.in (Corresponding Author)

Most cosmological models assume that the matter in the universe can be described by 'dust'(a pressure-less distribution) or at best a perfect fluid. Nevertheless, there is good reason to believe that - at least at the early stages of the universe - viscous effects do play a role.[27−29] For example, the existence of the bulk viscosity is equivalent to slow process of restoring equilibrium states.[30] The observed physical phenomena such as the large entropy per baryon and remarkable degree of isotropy of the cosmic microwave background radiation suggest analysis of dissipative effects in cosmology. Bulk viscosity is associated with the GUT phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn[31] for a review on cosmological models with bulk viscosity). The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity.[32−43]

Models with a relic cosmological constant Λ have received considerable attention recently among researchers for various reasons (see Refs.[44−48] and references therein). Some of the recent discussions on the cosmological constant "problem" and on cosmology with a time-varying cosmological constant by Ratra and Peebles,[49] Dolgov[50−52] and Sahni and Starobinsky[53] point out that in the absence of any interaction with matter or radiation, the cosmological constant remains a "constant", however, in the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time-varying Λ can be found. For these solutions, conservation of energy requires decrease in the energy density of the vacuum component to be compensated by a corresponding increase in the energy density of matter or radiation. Earlier researchers on this topic, are contained in Zeldovich,[54] Bertolami,[56,57] Weinberg,[45] and Carroll, Press and Turner.[55] Recent observations by Perlmutter *et al.*[58] and Riess *et al.*[59] strongly favour a significant and positive Λ. Their finding arise from the study of more than 50 type Ia supernovae with redshifts in the range $0.10 \leq z \leq 0.83$ and suggest Friedmann models with negative pressure matter such as a cosmological constant, domain walls or cosmic strings (Vilenkin,[60] Garnavich *et al.*[61]). Recently, Carmeli and Kuzmenko[62] have shown that the cosmological relativity theory (Behar and Carmeli[63]) predicts the value $\Lambda = 1.934 \times 10^{-35} s^{-2}$ for the cosmological constant. This value of Λ is in excellent agreement with the measurements recently obtained by the High-Z Supernova Team and Supernova Cosmological Project (Garnavich *et al.*;[61] Perlmutter *et al.*;[58] Riess *et al.*;[59] Schmidt *et al.*[64]). The main conclusion of these works is that the expansion of the universe is accelerating.

Several ans*ä*tz have been proposed in which the Λ term decays with time (see Refs. Gasperini,[65,66] Berman,[67] Freese *et al.*,[48] Özer and Taha,[48] Peebles and Ratra,[68] Chen and Hu,[69] Abdussattar and Viswakarma,[70] Gariel and Le Denmat,[71] Pradhan *et al.*[45,72]). Of the special interest is the ansatz $\Lambda \propto S^{-2}$ (where S is the scale factor of the Robertson-Walker metric) by Chen and Wu,[69] which has been considered/modified by several authors

(Abdel-Rahaman,[73] Carvalho *et al.*,[48] Waga,[74] Silveira and Waga,[48] Pradhan and Pandey,[75] Vishwakarma[76]).

Recently Bali and Gokhroo[77] obtained a Bianchi type I anisotropic magnetized cosmological model for perfect fluid distribution. Motivated by the situations discussed above, in this paper, we shall focus upon the problem of establishing a formalism for studying the general relativistic evolution of magnetic homogeneities in presence of bulk viscous in an expanding universe. We do this by extending the work of Bali and Gokhroo [77] by including an electrically neutral bulk viscous fluid as the source of matter in the energy-momentum tensor. This paper is organised as follows. The metric and the field equations are presented in Sec.2. The Sec.3 includes the solution of the field equations and physical and geometrical features of the models. In Sec.4, we obtain the bulk viscous cosmological solutions in the absence of the magnetic field. In Sec. 5, we discuss our main results and summarize the conclusions.

2. Field Equations

We consider the Bianchi type I metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2, \tag{1}$$

where A, B and C are functions of t only. The energy momentum tensor in the presence of bulk stress has the form

$$T_i^j = (\rho + \bar{p})v_i v^j + \bar{p}g_i^j + E_i^j, \tag{2}$$

where E_i^j is the electro-magnetic field given by Lichnerowicz[78] as

$$E_i^j = \bar{\mu}\left[|h|^2\left(v_i v^j + \frac{1}{2}g_i^j\right) - h_i h^j\right] \tag{3}$$

and

$$\bar{p} = p - \xi v_{;i}^i \tag{4}$$

Here ρ, p, $\bar{p}$ and ξ are the energy density, isotropic pressure , effective pressure, bulk viscous coefficient respectively and v^i is the flow vector satisfying the relation

$$g_{ij}v^i v^j = -1 \tag{5}$$

$\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}} \, {}^*F_{ji}v^j \tag{6}$$

where ${}^*F_{ij}$ is the dual electro-magnetic field tensor defined by Synge [79] to be

$$ {}^*F_{ij} = \frac{\sqrt{-g}}{2}\epsilon_{ijkl}F^{kl} \tag{7}$$

F_{ij} is the electro-magnetic field tensor and ϵ_{ijkl} is the Levi-Civita tensor density. Here, the comoving coordinates are taken to be $v^1 = 0 = v^2 = v^3$ and $v^4 = 1$. We take the incident magnetic field to be in the direction of x-axis so that $h_1 \neq 0$, $h_2 = 0 = h_3 = h_4$. Due to assumption of infinite conductivity of the fluid, we get $F_{14} = 0 = F_{24} = F_{34}$. The only non- vanishing component of F_{ij} is F_{23}. The first set of Maxwell's equation

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \tag{8}$$

leads to

$$F_{23} = I(\text{const.}) \tag{9}$$

where the semicolon represents a covariant differentiation. Hence

$$h_1 = \frac{AI}{\bar{\mu}BC} \tag{10}$$

The Einstein's field equations

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -8\pi T_i^j, \quad (\text{c = 1, G = 1 in gravitational unit}) \tag{11}$$

for the line element (1) has been set up as

$$8\pi \left(\bar{p} - \frac{I^2}{2\bar{\mu}B^2C^2} \right) = -\frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{B_4C_4}{BC} - \Lambda \tag{12}$$

$$8\pi \left(\bar{p} + \frac{I^2}{2\bar{\mu}B^2C^2} \right) = -\frac{A_{44}}{A} - \frac{C_{44}}{C} - \frac{A_4C_4}{AC} - \Lambda \tag{13}$$

$$8\pi \left(\bar{p} + \frac{I^2}{2\bar{\mu}B^2C^2} \right) = -\frac{A_{44}}{A} - \frac{B_{44}}{B} - \frac{A_4B_4}{AB} - \Lambda \tag{14}$$

$$8\pi \left(\rho + \frac{I^2}{2\bar{\mu}B^2C^2} \right) = \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} + \Lambda \tag{15}$$

The suffix 4 by the symbols A, B and C denote differentiation with respect to t.

3. Solution of the field equations

Equations (12) - (15) represent a system of four equations in six unknowns A, B, C, p, ρ and Λ. To get a determinate solution, we need two extra conditions. First we assume

$$A = BC \tag{16}$$

From Eqs. (13) and (14), we get

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} = \frac{A_4}{A} \left(\frac{C_4}{C} - \frac{B_4}{B} \right) \tag{17}$$

which leads to

$$\frac{\nu_4}{\nu} = \frac{\alpha}{\epsilon^2} \tag{18}$$

where $BC = \epsilon$, $\frac{B}{C} = \nu$, and α is a constant of integration. From Eqs. (12) and (13), we get

$$\frac{K}{B^2 C^2} = \frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{A_4 C_4}{AC} - \frac{B_4 C_4}{BC} \qquad (19)$$

where

$$K = -\frac{8\pi I^2}{\bar{\mu}} \qquad (20)$$

By using the condition (16) in (19), we have

$$\frac{C_{44}}{C} + \frac{2B_4 C_4}{BC} + \frac{C_4^2}{C^2} = \frac{K}{B^2 C^2} \qquad (21)$$

which leads to

$$\frac{\epsilon_{44}}{2\epsilon} + \frac{\epsilon_4^2}{2\epsilon^2} = \frac{K}{\epsilon^2} \qquad (22)$$

Setting $\epsilon_4 = f(\epsilon)$ in Eq. (22), we have

$$2\epsilon f f^1 + 2f^2 = 4K \qquad (23)$$

which reduces to

$$\frac{d}{d\epsilon}(f^2) + \frac{2f^2}{\epsilon} = \frac{4K}{\epsilon} \qquad (24)$$

Integrating Eq. (24) leads to

$$f^2 = \frac{(2K\epsilon^2 + N)}{\epsilon^2}, \qquad (25)$$

where N is the constant of integration. Eq. (25) on integration leads to

$$\sqrt{(2K\epsilon^2 + N)} = 2Kt + L, \qquad (26)$$

where L is the integrating constant. Eq. (26) can be rewritten as

$$\epsilon = \sqrt{(at^2 + bt + d)}, \qquad (27)$$

where

$$a = 2K; \;\; b = 2L \;\; and \;\; d = \frac{(L^2 - N)}{2K}. \qquad (28)$$

From Eq. (18), we get

$$\frac{\nu_4}{\nu} = \frac{\alpha}{(at^2 + bt + d)} \qquad (29)$$

Integrating Eq. (29) leads to

$$\nu = \frac{1}{M} e^{\frac{2\alpha}{\sqrt{(4ad - b^2)}} \, tan^{-1} \frac{(2at + b)}{\sqrt{(4ad - b^2)}}}, \;\; if \;\; 4ad - b^2 > 0 \qquad (30)$$

where M is a constant of integration.

$$\nu = L \left[\frac{(t + \frac{b}{2a}) - \frac{\sqrt{l}}{2a}}{(t + \frac{b}{2a}) - \frac{\sqrt{l}}{2a}} \right]^{\frac{\alpha}{\sqrt{l}}}, \;\; if \;\; 4ad - b^2 < 0 \qquad (31)$$

where $l > 0$.

$$\nu = e^{-\frac{\alpha}{a(t+\frac{b}{2a})}}, \ \ if \ \ 4ad - b^2 = 0 \tag{32}$$

3.1. Case (i) : when $4ad - b^2 > 0$.

In this case, we have

$$A^2 = at^2 + bt + d \tag{33}$$

$$B^2 = \frac{\sqrt{(at^2 + bt + d)}}{M} e^{\frac{2\alpha}{\sqrt{(4ad-b^2)}} tan^{-1}\frac{(2at+b)}{\sqrt{(4ad-b^2)}}} \tag{34}$$

$$C^2 = \frac{M\sqrt{(at^2 + bt + d)}}{\left[e^{\frac{2\alpha}{\sqrt{(4ad-b^2)}} tan^{-1}\frac{(2at+b)}{\sqrt{(4ad-b^2)}}} \right]} \tag{35}$$

Hence, the geometry of the spacetime (1) reduces to the form

$$ds^2 = -dt^2 + (at^2 + bt + d)dx^2$$

$$+ \frac{\sqrt{(at^2 + bt + d)}}{M} e^{\frac{2\alpha}{\sqrt{(4ad-b^2)}} tan^{-1}\frac{(2at+b)}{\sqrt{(4ad-b^2)}}} dy^2$$

$$+ \frac{M\sqrt{(at^2 + bt + d)}}{\left[e^{\frac{2\alpha}{\sqrt{(4ad-b^2)}} tan^{-1}\frac{(2at+b)}{\sqrt{(4ad-b^2)}}} \right]} dz^2 \tag{36}$$

The effective pressure $\bar{p}$ and density ρ for the model (36) are given by

$$8\pi(p - \xi\theta) = \frac{4(b^2 - 4ad) + (2at + b)^2 - 4\alpha^2}{16(at^2 + bt + d)^2} - \frac{K}{2(at^2 + bt + d)} - \Lambda \tag{37}$$

$$8\pi\rho = \frac{5(2at + b)^2 - 4\alpha^2}{16(at^2 + bt + d)^2} + \frac{K}{2(at^2 + bt + d)} + \Lambda \tag{38}$$

where θ is the scalar of expansion calculated for the flow vector v^i and is given by

$$\theta = \frac{(2at + b)}{(at^2 + bt + d)} \tag{39}$$

For the specification of ξ, now we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{40}$$

where $\gamma(0 \le \gamma \le 1)$ is a constant.

Thus, given $\xi(t)$ we can solve the cosmological parameters. In most of the investigations involving bulk viscosity is assumed to be a simple power function of the energy density.[32−34]

$$\xi(t) = \xi_0 \rho^n, \tag{41}$$

where ξ_0 and n are constants. If $n = 1$, Eq. (41) may correspond to a radiative fluid.[35] However, more realistic models[36] are based on lying in the regime $0 \leq n \leq \frac{1}{2}$. On using Eq. (41) in Eq. (37), we obtain

$$8\pi(p - \xi_0 \rho^n \theta) = \frac{4(b^2 - 4ad) + (2at + b)^2 - 4\alpha^2}{16T^2} - \frac{K}{2T} - \Lambda \tag{42}$$

where $T = at^2 + bt + d$.

3.1.1. *Model (i)* : $(\xi = \xi_0)$.

When $n = 0$, Eq. (41) reduces to $\xi = \xi_0$. With the use of Eqs. (38), (39) and (40), Eq. (42) reduces to

$$8\pi\gamma\rho = \frac{8\pi(2at + b)\xi_0}{T} + \frac{4(b^2 - 4ad) + (2at + b)^2}{16T^2}$$

$$- \frac{K}{2T} - \Lambda \tag{43}$$

Eliminating $\rho(t)$ between Eqs. (38) and (43), we get

$$(1 + \gamma)\Lambda = \frac{1}{16T^2}\left[(2at + b)^2(1 - 5\gamma) + 4(b^2 - 4ad) + 4\gamma\alpha^2\right]$$

$$+ \frac{1}{2T}\left[K(1 - \gamma) + 16\pi\xi_0(2at + b)\right] \tag{44}$$

3.1.2. *Model (ii)* : $(\xi = \xi_0\rho)$.

When $n = 1$, Eq. (41) reduces to $\xi = \xi_0\rho$. Eq. (42), with the use of Eqs. (38), (39) and (40), reduces to

$$8\pi\rho\left[\gamma - \frac{(2at + b)\xi_0}{T^2}\right] = \frac{4(b^2 - 4ad) + (2at + b)^2 - 4\alpha^2}{16T^2}$$

$$- \frac{K}{2T} - \Lambda \tag{45}$$

Eliminating $\rho(t)$ between Eqs. (38) and (45), we get

$$\Lambda = \frac{1}{16\left[(1 + \gamma)T^2 - (2at + b)\xi_0\right]} \times \left[4\left(b^2 - 4ad\right) + (2at + b)^2 - \right.$$

$$4\alpha^2 - 8KT - \left(\gamma T^2 - (2at + b)\xi_0\right)$$

$$\left. \times \left\{\frac{5(2at + b)^2 - 4\alpha^2 + 8KT}{T^2}\right\}\right] \tag{46}$$

From Eqs. (44) and (46), we observe that the cosmological constant is a decreasing function of time and it approaches a small value as time progresses (i. e., the present epoch), which explains the small value of Λ at present. In the model (i), we also find that Λ is positive for

all values of γ lying in the interval $(0, \frac{1}{5})$ and $\alpha^2 > 5(4ad - b^2)$. In the model (ii), we see that the second term of the Eq. (46) decreases rapidly with expansion of the universe and the first term will dominate over the second term. Thus, from Eq. (46) it is also observed that Λ is positive when $K < \frac{a}{2}$ and $b^2 > 8Kd + 4\alpha^2$. These small positive values of Λ in both models (i) and (ii) are supported by results from recent supernovae observations (Perlmutter et al., [58] Riess et al., [59] Garnavich et al., [61] Schmidt et al. [64]).

Physical and Geometrical Features of the Models

The component of the shear tensor σ_i^j are given by

$$\sigma_1^1 = \frac{(2at + b)}{6T}$$

$$\sigma_2^2 = \frac{(2at + b)}{12T} + \frac{\alpha}{2T}$$

$$\sigma_3^3 = \frac{(2at + b)}{12T} - \frac{\alpha}{2T}$$

$$\sigma_4^4 = 0 \tag{47}$$

Thus

$$\sigma^2 = \frac{(2at + b)^2 - 12\alpha^2}{48T} \tag{48}$$

The rate of expansion H_i in the direction of x, y, z -axes are given by

$$H_1 = \frac{(2at + b)}{2T}$$

$$H_2 = \frac{(2at + b)}{4T} + \frac{\alpha}{2T}$$

$$H_3 = \frac{(2at + b)}{4T} - \frac{\alpha}{2T} \tag{49}$$

The model (36) starts expanding at $t > -\frac{b}{2a}$ and the expansion in the model decreases as time increases and stops at $t = \infty$. The model, in general, represents shearing and non-rotating universe. Since $\lim_{t \to \infty} \frac{\sigma}{\theta} \neq 0$, therefore, the model does not approach isotropy for large values of t. There is a singularity in the model at $t = 0$ which is real physical singularity.

3.2. Case (ii) :   *when* $4ad - b^2 < 0$ *i.e.* $4ad - b^2 = -l$, $l > 0$.

In this case the geometry of the spacetime (1) takes the form

$$ds^2 = -dt^2 + a \left[(t + \frac{b}{2a})^2 - \frac{l}{4a^2} \right] dx^2$$

$$+ a \left[(t + \frac{b}{2a})^2 - \frac{l}{4a^2} \right]^{\frac{1}{2}} \left[\frac{(t + \frac{b}{2a}) - \frac{\sqrt{l}}{2a}}{(t + \frac{b}{2a}) + \frac{\sqrt{l}}{2a}} \right]^{\frac{\alpha}{\sqrt{l}}} dy^2$$

$$+ a \left[(t + \frac{b}{2a})^2 - \frac{l}{4a^2} \right]^{\frac{1}{2}} \left[\frac{(t + \frac{b}{2a}) - \frac{\sqrt{l}}{2a}}{(t + \frac{b}{2a}) + \frac{\sqrt{l}}{2a}} \right] dz^2 \tag{50}$$

The effective pressure and density for the model (50) are given by

$$8\pi(p - \xi\theta) = \frac{4a^2t^2 + 5b^2 - 4abt - 16ad}{16a^2T_1^2} - \frac{\alpha^2}{16T_1} - \frac{K}{2\bar{\mu}aT_1} - \Lambda \tag{51}$$

$$8\pi\rho = \frac{5(t + \frac{b}{2a})^2}{4T_1^2} - \frac{\alpha^2}{16T_1} - \frac{K}{2\bar{\mu}aT_1} + \Lambda \tag{52}$$

where the scalar expansion θ is given by

$$\theta = \frac{2(t + \frac{b}{2a})}{T_1} \tag{53}$$

and

$$T_1 = \left[(t + \frac{b}{2a})^2 - \frac{l}{4a^2} \right]$$

3.2.1. *Model (i):* $(\xi = \xi_0)$.

When $n = 0$, Eq. (41) reduces to $\xi = \xi_0$ and hence Eq. (51), with the use of Eqs. (40), (52) and (53), leades to

$$16\pi\gamma\rho = 16\pi\xi_0(t + \frac{b}{2a}) + \frac{1}{16a^2T_1^2} \left[4a^2t^2(1 + 5\gamma) + 5b^2(1 + \gamma) - 4at(1 - 5b\gamma) - 16ad \right]$$

$$- \frac{\alpha^2(1 - \gamma)}{16T_1} - \frac{K(1 + \gamma)}{2\bar{\mu}aT_1} - (1 - \gamma)\Lambda \tag{54}$$

Eliminating $\rho(t)$ between Eqs. (52) and (54), we get

$$(1 + \gamma)\Lambda = \frac{1}{16a^2T_1^2} \left[4a^2t^2(1 - 5\gamma) + 5b^2(1 - \gamma) - 4abt(1 + 5\gamma) - 16ad \right]$$

$$+ \frac{\alpha^2(1 - \gamma)}{16T_1} + \frac{K(1 - \gamma)}{2\bar{\mu}aT_1} - \frac{16\pi\xi_0(t + \frac{b}{2a})}{T_1} \tag{55}$$

3.2.2. *Model (ii) :* $(\xi = \xi_0\rho)$.

When $n = 1$, Eq. (41) reduces to $\xi = \xi_0\rho$. The Eq. (51), with the use of the Eqs. (40), (52) and (53), reduces to

$$8\pi\rho[a(1 + \gamma)T_1 - (2at + b)\xi_0] = \frac{4a^2t^2 + 5b^2 - 4abt - 16ad}{16aT_1} +$$

$$\frac{5(2at + b)^2}{16aT_1} - \frac{a\alpha^2}{8} - \frac{K}{\bar{\mu}} \tag{56}$$

Eliminating $\rho(t)$ between Eqs. (52) and (54), we get

$$\Lambda[a(1+\gamma)T_1 - (2at+b)\xi_0] = \frac{4a^2t^2 + 5b^2 - 4abt - 16ad}{16aT_1} - \frac{a\alpha^2}{16} - \frac{K}{2\bar{\mu}}$$

$$- a\gamma T_1 - (2at+b)\xi_0 \left[\frac{5(t+\frac{b}{2a})^2}{4T_1^2} - \frac{\alpha^2}{16T_1^2} - \frac{K}{2\bar{\mu}aT_1} \right] \tag{57}$$

In both models (i) and (ii), we have observed from Eqs. (55) and (57) that the cosmological constant is a decreasing function of time and it approaches a small value as time progresses (i.e., the present epoch), which explains the small value of Λ at present.

The Physical and Geometrical Features of the Models

The non-vanishing components of shear tensor (σ_i^j) are given by

$$\sigma_1^1 = \frac{2(t+\frac{b}{2a})}{(t+\frac{b}{2a})^2 - \frac{l}{4a^2}} \tag{58}$$

$$\sigma_2^2 = \frac{1}{12} \left[\frac{3\alpha}{(t+\frac{b}{2a})^2 - \frac{l}{4a^2}} - \frac{2(t+\frac{b}{2a})}{(t+\frac{b}{2a})^2 - \frac{l}{4a^2}} \right] \tag{59}$$

$$\sigma_3^3 = -\frac{1}{12} \left[\frac{3\alpha}{(t+\frac{b}{2a})^2 - \frac{l}{4a^2}} + \frac{2(t+\frac{b}{2a})}{(t+\frac{b}{2a})^2 - \frac{l}{4a^2}} \right] \tag{60}$$

The models start expanding with a big bang at $t = -\frac{b}{2a} + \frac{\sqrt{l}}{2a}$ and the expansion in the models decreases with time. The expansion in the models stops at $t = \infty$. Since $lim_{t\to\infty}\frac{\sigma}{\theta} \neq 0$. Therefore, the models do not approach isotropy for large values of t.

 3.3. Case (iii) :   when $4ad - b^2 = 0$.

In this case the metric (1) reduces to

$$ds^2 = -dT_2^2 + T_2^2\, dX^2 + T_2\, e^{-\frac{\alpha}{aT_2}}\, dY^2 + T_2\, e^{\frac{\alpha}{aT_2}}\, dZ^2, \tag{61}$$

where

$$T_2 = t + \frac{b}{2a}$$

The effective pressure $\bar{p}$ and density ρ for the model (61) are given by

$$8\pi\bar{p} = 8\pi(p - \xi\theta) = \frac{1}{4T_2^2} - \frac{\alpha^2}{4a^2T_2^4} - \frac{K}{2aT_2^2} - \Lambda, \tag{62}$$

$$8\pi\rho = \frac{5}{4T_2^2} - \frac{\alpha^2}{4a^2T_2^4} + \frac{K}{2aT_2^2} + \Lambda, \tag{63}$$

where the scalar expansion θ is given by

$$\theta = \frac{2}{T_2} \tag{64}$$

3.3.1. *Model (i):*   ($\xi = \xi_0$).

When $n = 0$, Eq. (41) gives $\xi = \xi_0$ and hence Eq. (62), with the use of Eqs. (40), (63) and (64), reduces to

$$\rho = \frac{1}{8\pi(1+\gamma)T_2}\left[16\pi\xi_0 + \frac{3}{2T_2} - \frac{\alpha^2}{2a^2T_2^3}\right]$$
(65)

Eliminating $\rho(t)$ between Eqs. (63) and (65), we get

$$(1+\gamma)\Lambda = \frac{16\pi\xi_0}{T_2} + \frac{1}{4aT_2^2}\left[a(1-5\gamma) - 2(1+\gamma)K\right] - \frac{(1-\gamma)\alpha^2}{4a^2T_2^4}$$
(66)

3.3.2. *Model (ii) :*   ($\xi = \xi_0\rho$).

When $n = 1$, Eq. (41) gives $\xi = \xi_0\rho$ and hence Eq. (62), with the use of Eqs. (40), (63) and (64), reduces to

$$\rho = \frac{1}{8\pi[(1+\gamma)T_2 - 2\xi_0]}\left[\frac{3}{2T_2} - \frac{\alpha^2}{2a^2T_2^3}\right]$$
(67)

Eliminating $\rho(t)$ between Eqs. (63) and (67), we get

$$\Lambda = \frac{1}{\{(1+\gamma)T_2 - 2\xi_0\}T_2}\left[\frac{(1-5\gamma)}{4} - \frac{(1+\gamma)K}{2a} - \frac{(1-\gamma)\alpha^2}{4a^2T_2^2} + \right.$$
$$\left. \frac{2\xi_0}{T_2}\left\{\frac{5}{4} + \frac{K}{2a} - \frac{\alpha^2}{4a^2T_2^2}\right\}\right]$$
(68)

From Eqs. (66) and (68), we observe that the cosmological constant is a decreasing function of time and it approaches a small value as time progresses (i. e., the present epoch), which explains the small value of Λ at present. In the model (i), from Eq. (66), it can be seen that first and second terms dominate over the third term. Further we also find that Λ is positive for all values of γ lying in the interval $(0, \frac{1}{5})$ and $K < \frac{a(1-5\gamma)}{2(1+\gamma)}$. In the model (ii), from Eq. (68), one can easily see that that for all value of γ lying in the interval $(0, \frac{1}{5})$ and $\xi_0 < \frac{(1+\gamma)b}{4a}$, $K < \frac{a(1-5\gamma)}{2(1+\gamma)}$, Λ is positive. These small positive values of Λ are consistent by the results from recent supernovae observations.

The Physical and Geometrical Features of the Models

The non-vanishing components of shear tensor (σ_i^j) are given by

$$\sigma_1^1 = \frac{1}{3T_2}$$
(69)

$$\sigma_2^2 = \frac{1}{6}\left[\frac{3\alpha}{aT_2^2} - \frac{1}{T_2}\right]$$
(70)

$$\sigma_2^2 = -\frac{1}{6}\left[\frac{3\alpha}{aT_2^2} + \frac{1}{T_2}\right]$$
(71)

The models start expanding with a big bang $t = -\frac{b}{2a}$ and the expansion in the models decreases with time increase. The expansion in the models stop at $t = \infty$. Since $lim_{t \to \infty} \frac{\sigma}{\theta} \neq 0$. Therefore, the models do not approach isotropy for large values of t.

4. Solution of the field equations in absence of magnetic field

In the absence of magnetic field, the metric (36) reduces to

$$ds^2 = -dt^2 + (bt + \beta)dx^2 + \frac{l\sqrt{(bt + \beta)}}{M}dy^2 + \frac{M\sqrt{(bt + \beta)}}{l}dz^2, \tag{72}$$

where

$$l = e^{\frac{2\alpha}{\sqrt{h - b^2}} tan^{-1} \frac{b}{\sqrt{h - b^2}}}; \quad a = 2K$$

$$d = \frac{L^2 - N}{2K} = \beta \frac{\sin 2K}{2K}; \quad 4ad = 4(L^2 - N) = h \tag{73}$$

Thus when $K \to 0$ then $d \to \beta$. In absence of the magnetic field i.e. when $K \to 0$ then effective pressure $\bar{p}$ and density ρ for the model (72) are given by

$$8\pi\bar{p} = 8\pi(p - \xi\theta) = \frac{b^2 - d}{4(bt + \beta)^2} + \frac{b^2}{16(bt + \beta)^2} - \frac{\alpha^2}{4(bt + \beta)^2} - \Lambda, \tag{74}$$

$$8\pi\rho = \frac{5b^2}{16(bt + \beta)^2} - \frac{\alpha^2}{4(bt + \beta)^2} + \Lambda, \tag{75}$$

where scalar expansion θ is given by

$$\theta = \frac{b}{bt + \beta} \tag{76}$$

4.1. *Model (i):* $(\xi = \xi_0)$.

When $n = 0$, Eq. (41) gives $\xi = \xi_0$ and hence Eq. (74), with the use of Eqs. (40), (75) and (76), reduces to

$$8\pi(1 + \gamma)\rho = \frac{8\pi b\xi_0}{(bt + \beta)} + \frac{5b^2 - 4\alpha^2 - 2d}{8(bt + \beta)^2} \tag{77}$$

Eliminating $\rho(t)$ between Eqs. (75) and (77), we get

$$(1 + \gamma)\Lambda = \frac{8\pi b\xi_0}{(bt + \beta)} - \frac{d}{4(bt + \beta)^2} \tag{78}$$

4.2. *Model (ii) :* $(\xi = \xi_0\rho)$.

When $n = 1$, Eq. (41) reduces to $\xi = \xi_0\rho$. Eq. (74), with the use of Eqs. (40), (75), and (76), reduces to

$$8\pi\rho = \frac{(5b^2 - 4\alpha^2 - 2d)}{8(bt + \beta)[(1 + \gamma)(bt + \beta) - b\xi_0]} \tag{79}$$

Eliminating $\rho(t)$ between Eqs. (75) and (79), we get

$$\Lambda = \frac{\left[(1-\gamma)(5b^2 - 4\alpha^2) - 4ad + \frac{(5b^2 - 4\alpha^2)b\xi_0}{(bt+\beta)}\right]}{16(bt+\beta)\left[(1+\gamma)(bt+\beta) - b\xi_0\right]} \qquad (80)$$

In both the models (i) and (ii), from Eqs. (78) and (80), we observe that the cosmological constant is a decreasing function of time and it approaches a small value as time progresses (i. e., the present epoch), which explains the small value of Λ at present. In the model (i), from Eq. (78), it can be seen that for all time $t > \frac{d}{32\pi b^2 \xi_0} - \frac{\beta}{B}$, the cosmological "constant" is positive definite. In model (ii), Eq. (80) suggest that the positivity of Λ demands the relation among constant as $b^2 > \frac{4}{5}(\alpha^2 + ad)$ and $\xi_0 < \frac{(1+\gamma)\beta}{b}$. These small positive values of Λ in both models are supported by the results from recent supernovae observations.
The Physical and Geometrical Features of the Models.
In the absence of magnetic field, the shear (σ_i^j) are given by

$$\sigma^2 = \frac{b^2 + 12\alpha^2}{48(bt+\beta)^2} \qquad (81)$$

Here, $lim_{t\to\infty} \frac{\sigma}{\theta} \neq 0$. The model does not approach isotropy for large values of t in absence of magnetic field also.

5. Conclusions

We have obtained a new class of Bianchi type I anisotropic magnetized cosmological models with a bulk viscous fluid as the source of matter. Generally, the models are expanding, shearing and non-rotating. In all these models, we observe that they do not approach isotropy for large values of time t either in the presence or in the absence of magnetic field.
The cosmological constant in all models given in Sec. 3 and Sec. 4 are decreasing function of time and they all approach a small value as time increases (i.e., the present epoch). The values of cosmological "constant" for these models are found to be small and positive which are supported by the results from recent supernovae observations recently obtained by the High - Z Supernova Team and Supernova Cosmological Project (Garnavich *et al.*;[61] Perlmutter *et al.*;[58] Riess *et al.*;[59] Schmidt *et al.* [64]). Thus, with our approach, we obtain a physically relevant decay law for the cosmological constant unlike other investigators where *adhoc* laws were used to arrive at a mathematical expressions for the decaying vacuum energy. Thus our models are more general than those studied earlier.

Acknowledgements

References

[1] Ya. B. Zeldovich, A. A. Ruzmainkin and D. D. Sokoloff, *Magnetic field in Astrophysics*, (New York, Gordon and Breach, 1993).

[2] E. R. Horrison, *Phys. Rev. Lett.* **30**, 188 (1973).

[3] H. P. Robertson and A. G. Walker, *Proc. London Math. Soc.* **42**, 90 (1936).

[4] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (W. H. Freeman, New York, 1973).

[5] E. Asseo and H. Sol, *Phys. Rep.* **6**, 148 (1987).

[6] M. A. Melvin, *Ann. New York Acad. Sci.* **262**, 253 (1975).

[7] R. Pudritz and J. Silk, *Astrophys. J.* **342**, 650 (1989).

[8] K. T. Kim, P. G. Tribble and P. P. Kronberg, *Astrophys. J.* **379**, 80 (1991).

[9] R. Perley and G. Taylor, *Astrophys. J.* **101**, 1623 (1991).

[10] P. P. Kronberg, J. J. Perry and E. L. Zukowski, *Astrophys. J.* **387**, 528 (1991).

[11] A. M. Wolfe, K. Lanzetta and A. L. Oren, *Astrophys. J.* **388**, 17 (1992).

[12] R. Kulsrud, R. Cen, J. P. Ostriker and D. Ryu, *Astrophys. J.* **380**, 481 (1997).

[13] E. G. Zweibel and C. Heiles, *Nature*, **385**, 131 (1997).

[14] J. D. Barrow, *Phys. Rev.* **D55**, 7451 (1997).

[15] Ya. A. Zeldovich, *Sov. Astron.* **13**, 608 (1970).

[16] M. S. Turner and L. M. Widrow, *Phys. Rev.* **D30**, 2743 (1988).

[17] J. Quashnock, A. Loeb and D. N. Spergel, *Astrophys. J.* **344**, L49 (1989).

[18] B. Ratra, *Astrophys. J.* **391**, L1 (1992).

[19] A. D. Dolgov and J. Silk, *Phys. Rev.* **D47**, 3144 (1993)

[20] A. D. Dolgov, *Phys. Rev.* **D48**, 2499 (1993)

[21] R. Bali, *Int. J. Theor. Phys.* **47**, 7 (1986);
R. Bali and M. Ali, *Pramana*, **47**, 25 (1996).

[22] K. S. Thorne, *Astrophys. J.* **148**, 51 (1967).

[23] K. C. Jacobs, *Astrophys. J.* **153**, 661 (1968).

[24] K. C. Jacobs, *Astrophys. J.* **155**, 379 (1969).

[25] S. R. Roy, S. Narain and J. P. Singh, *Austr. J. Phys.* **2**, 239 (1985).

[26] C. G. Tsagas and R. Maartens, gr-qc/9912022 (2000).

[27] W. Israel and J. N. Vardalas, *Lett. nuovo Cim.* **4**, 887 (1970).

[28] Z. Klimek, *Post. Astron.* **19**, 165 (1971).

[29] S. Weinberge, *Astrophys. J.* **168**, 175 (1971).

[30] L. Landau and E. M. Lifshitz, "Fluid Mechanics", Addison-Wisley, Mass. 1962, p. 304.

[31] Ø. Grøn, *Astrophys. Space Sc.* **173**, 191 (1990).

[32] D. Pavon, J. Bafaluy and D. Jou, *Class Quantum Gravit.* **8**, 357 (1991); "Proc. Hanno Rund Conf. on Relativity and Thermodynamics", Ed. S. D. Maharaj, University of Natal, Durban, (1996), p. 21.

[33] R. Maartens, *Class Quantum Gravit.* **12**, 1455 (1995).

[34] W. Zimdahl, *Phys. Rev.* D **53**, 5483 (1996).

[35] S. Weinberg, *Gravitation and Cosmology*, (Wiley, New York, 1972), p. 57.

[36] N. O. Santos, R. S. Dias and A. Banerjee, *J. Math. Phys.* **26**, 878 (1985).

[37] T. Padmanabhan and S. M. Chitre, *Phys. Lett.* **A120**, 433 (1987).

[38] V. B. Johri and R. Sudarshan, *Phys. Lett.* **A132**, 316 (1988).

[39] A. Pradhan, R. V. Sarayakar and A. Beesham, *Astr. Lett. Commun.* **35**, 283 (1997).

[40] A. Pradhan, V. K. Yadav and I. Chakrabarty, *Int. J. Mod. Phys.* **D10**, 339 (2001).

[41] I. Chakrabarty, A. Pradhan and N. N. Saste, *Int. J. Mod. Phys.* **D10**, 741 (2001).

[42] A. Pradhan and V. K. Yadav, *Int. J. Mod. Phys.* **D11**, 857 (2002).

[43] A. Pradhan and I. Aotemshi, *Int. J. Mod. Phys.* **D11**, 1419 (2002).

[44] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).

[45] A. Pradhan and A. Kumar, *Int. J. Mod. Phys.* **D10**, 291 (2001)..

[46] J. A. Frieman and I. Waga, *Phys. Rev.* **D57**, 4642 (1998).

[47] R. Carlberg, *et al*. *Astrophys. J.* **462**, 32 (1996).

[48] M. Özer and M. O. Taha, *Nucl. Phys.* **B287**, 776 (1987);
K. Freese, F. C. Adams, J. A. Frieman and E. Motta, *ibid.* B **287**, 1797 (1987);
J. C. Carvalho, J. A. S. Lima and I. Waga, *Phys. Rev.* **D46**, 2404 (1992);
V. Silviera and I. Waga., *ibid.* **D50**, 4890 (1994); *Phys. Rev.* **D50**, 4890 (1994).

[49] B. Ratra and P. J. E. Peebles, *Phys. Rev.* **D37**, 3406 (1988).

[50] A. D. Dolgov, in *The Very Early Universe*, eds. G. W. Gibbons, S. W. Hawking and S. T. C. Siklos (Cambridge University Press, 1983).

[51] A. D. Dolgov, M. V. Sazhin and Ya. B. Zeldovich, *Basics of Modern Cosmology* (Editions Frontiers, 1990).

[52] A. D. Dolgov, *Phys. Rev.* **D55**, 5881 (1997).

[53] V. Sahni and A. Starobinsky, *Int. J. Mod. Phys.* **D9**, 373 (2000); gr-qc/9904398 (2000).

[54] Ya. B. Zeldovich, *Sov. Phys.-Uspekhi* **11**, 381 (1968).

[55] S. M. Carroll, W. H. Press and E. L. Turner, *Ann. Rev. Astron. Astrophys.* **30**, 499 (1992).

[56] O. Bertolami, *Nuovo Cimento* **93**, 36 (1986).

[57] O. Bertolami, *Fortschr. Phys.* **34**, 829 (1986).

[58] S. Perlmutter *et al.*, *Astrophys. J.* **483**, 565 (1997), Supernova Cosmology Project Collaboration (astro-ph/9608192); *Nature* **391**, 51 (1998), Supernova Cosmology Project Collaboration (astro-ph/9712212); *Astrophys. J.* **517**, 565 (1999), Project Collaboration (astro-ph/9608192).

[59] A. G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998); Hi-Z Supernova Team Collaboration (astro-ph/9805201).

[60] A. Vilenkin, *Phys. Rep.* **121**, 265 (1985).

[61] P. M. Garnavich *et al.*, *Astrophys. J.* **493**, L53 (1998a), Hi-Z Supernova Team Collaboration (astro-ph/9710123); *Astrophys. J.* **509**, 74 (1998b); Hi-Z Supernova Team Collaboration (astro-ph/9806396).

[62] M. Carmeli and T. Kuzmenko, *Int. J. Theor. Phys.* **41**, 131 (2002).

[63] S. Behar and M. Carmeli, *Int. J. Theor. Phys.* **39**, 1375 (2002); astro-ph/0008352.

[64] B. P. Schmidt *et al.*, *Astrophys. J.* **507**, 46 (1998), Hi-Z Supernova Team Collaboration (astro-ph/9805200).

[65] M. Gasperini, *Phys. Lett.* **B194**, 347 (1987).

[66] M. Gasperini, *Class. Quantum Grav.* **5**, 521 (1988).

[67] M. S. Berman, *Int. J. Theor. Phys.* **29**, 567 (1990); *Int. J. Theor. Phys.* **29**, 1419 (1990);
Phys. Rev. **D43**, 75 (1991);
M. S. Berman and M. M. Som, *Int. J. Theor. Phys.* **29**, 1411 (1990);
M. S. Berman, M. M. Som and F. M. Gomide, *Gen. Rel. Gravit.* **21**, 287 (1989);
M. S. Berman and F. M. Gomide, *Gen. Rel. Gravit.* **22**, 625 (1990).

[68] P. J. E. Peebles and B. Ratra, *Astrophys. J.* **325**, L17 (1988).

[69] W. Chen and Y. S. Wu, *Phys. Rev.* **D41**, 695 (1990).

[70] Abdussattar and R. G. Vishwakarma, *Pramana J. Phys.* **47**, 41 (1996).

[71] J. Gariel and G. Le Denmat, *Class. Quant. Grav.* **16**, 149 (1999).

[72] A. Pradhan and V. K. Yadav, *Int. J. Mod. Phys.* **D11**, 839 (2002);
A. Pradhan and H. R. Pandey, *Int. J. Mod. Phys.* **D**, in press; gr-qc/0211002 (2002).

[73] A.-M. M. Abdel-Rahaman, *Gen. Rel. Grav.* **22**, 655 (1990); *Phys. Rev.* **D45**, 3492 (1992).

[74] I. Waga, *Astrophys. J.* **414**, 436 (1993).

[75] A. Pradhan and O. P. Pandey, *Spacetime & Substance*, **14**, (2004), in press; gr-qc/0211008 (2002).

[76] R. G. Vishwakarma, *Class. Quantum Grav.* **17**, 3833 (2000).

[77] R. Bali and A. Gokhroo, *Astrophys. Space Sc.* **278**, 283 (2001).

[78] A. Lichnerowicz, *Relativistic Hydrodynamics and Magnetohydrodynamics* (W. A. Benjamin Inc., New York, 1967) p. 13.

[79] J. L. Synge, *Relativity: The General Theory* (North-Holland Publ. Co., Amsterdam, 1960), p. 356.